Why flux estimation is important using the technique like LETKF ?

- In several recent studies CH$_4$ flux has been estimated using inverse modeling that deals with the analytical solution to the Bayesian optimization problem.

- Most of these studies uses ground based observations, which are quite successful at delineating continental-scale flux variations at seasonal to decadal timescales, but lacked the temporal and spatial detail needed to study the carbon cycle at regional scale.

- Some investigations presented the smaller-scale inversion using satellite observations. But, those are generally performed at the annual scale.

- LETKF updates the surface flux at every grid and samples the observation at fine temporal resolution. In the present case sampling has been performed at every six hourly interval.
Analysis ensemble perturbation $X^a$ relaxed ($X^{a,RLX}$) to the prior spread using RTPS technique. The updated analyzed concentration ensembles are used as initial condition for the next ensemble forecast (Cycle2) while the analyzed surface fluxes ($F_1$) are used to force an ensemble $CH_4$ forecast throughout the data assimilation cycle of Cycle2.

In the LETKF, the analysis ensemble mean $\tilde{x}^a$ is derived using background ensemble mean $\tilde{x}^b$ and ensemble perturbations $X^b$.

Figure 1: The dashed lines show the ensemble forecast of $CH_4$ concentrations, the solid line shows the linear combination of the forecasts, the filled circles show the observations of $CH_4$. The data assimilation finds the linear combination of the ensemble forecast by estimating the $w^a$ that best fits the observations throughout the assimilation window (schematic adapted from Kalnay (2010); Miyazaki et al. 2010).
The Ensemble Kalman filter (EnKF) needs variance inflation to mitigate the under dispersive ensemble. The EnKF estimates the background error covariance $P^b$ by an ensemble:

$$ P^b = \frac{1}{K-1} X^b [X^b]^T $$

**Multiplicative inflation:**
The multiplicative method inflates the prior ensemble such as:

$$ P^b_{\text{infl}} = \gamma P^b $$

**Relaxation to prior spread (RTPS):**
The relaxation-to-prior spread method relaxes the reduction of the ensemble spread after relaxing the analysis ensemble spread to the background(prior) ensemble spread (Whitaker and Hamill (2012)) such as:

$$ X^a_{\text{infl}} = \left( \frac{\alpha_{\text{RTPS}} \sigma^b + (1-\alpha_{\text{RTPS}}) \sigma^a}{\sigma^a} \right) X^a_{\text{tmp}} $$

where $\alpha_{\text{RTPS}}$ and $\sigma$ denote the relaxation parameter of the RTPS and the ensemble spread, respectively.
Experimental Settings

- Forward simulations with MIROC4-ACTM
  - **Horizontal resolution**: T42 spectral truncations (~2.8° x 2.8° latitude-longitude grid)
  - **Pressure levels**: hybrid vertical coordinate of 67 levels from Earth’s surface to 0.0128 hPa
  - 100 ensemble members
  - **Covariance inflation**: Fixed multiplicative inflation and RTPS inflation method
  - **Assimilation window**: 8 days
  - An initial perturbation with approximately 10% uncertainty in the a priori flux is given that are generated from random positive values with normal distribution

- Emissions
  - **Anthropogenic**: the anthropogenic emissions are used from the Emission Database for Global Atmospheric Research, version 4.3.2 inventory (EDGARv4.3.2) (Janssens-Maenhout et al. 2019) (includes major sectors such as; fugitive, enteric fermentation and manure management, solid waste and wastewater handling).
  - **Biomass burning**: The biomass burning emissions are taken from the Global Fire Database (GFEDv4s) (Van der Werf et al. 2017) and Goddard Institute for Space Studies emissions (Fung et al. 1991).
  - **Wetland and rice**: the wetland and rice emissions are taken from the process-based model of the terrestrial biogeochemical cycle, Vegetation Integrated Simulator of Trace gases (VISIT) (Ito et al. 2019) that is based on Cao et al. (1996).
  - **Other emission**: the other emissions such as; ocean, termites, mud volcano etc. are taken from TransCom-CH₄ inter-comparison experiment (Patra et al. 2011).

- Following two experiments have been performed:
  - **Experiment1**:
    - CH4 fluxes as mentioned are used as “true” fluxes in generating simulated observations (3D CH₄ concentration) in Observation System Simulation Experiment (OSSE).
    - Only surface layer of CH₄ concentration is used at each grid.
    - The true fluxes are reproduced by reducing the prior flux by 30% (the uncertainty in regional flux estimation is found to be at 30% or lower (Chandra et al., 2021)). Prior fluxes have same seasonal cycles as true fluxes.
  - **Experiment2**:
    - In another OSSE experiments, the synthetic column CH₄ mixing ratios are assimilated with a coverage mimicking GOSAT satellite tracks over Asia.
Figure 2: (a) Spatial plot in the first column shows the true CH₄ flux during Feb 2010, bottom Figure in the same column shows the bias in the prior flux. Second column shows the true flux estimation based on multiplicative inflation and the estimated flux bias (bottom Figure in same column) and the Figure in the third column shows the flux estimation based on RTPS inflation method and the estimated flux bias (bottom Figure in same column). (b) Same as (a) but during July 2010.

Figure 2 and Figure 3 show that, both multiplicative and RTPS techniques are able to retrieve the true fluxes largely during both seasons. However, RTPS works better than multiplicative inflation throughout the year.

Figure 3: Total sum of the CH₄ fluxes over landmass region shown in Figure 2.
Experiment 2: CH$_4$ Flux estimation using column averaged CH$_4$ over East and Southeast Asia (OSSE) and summary

- Column averaged OSSE derived as follows:
  - Column averaged CH$_4$ from ACTM spatially collocated with GOSAT using following equation.
    \[
    X_{\text{ACTM}} = X_{\text{a priori}} + AK(X_{\text{ACTM}} - X_{\text{a priori}})
    \]
  - where $X_{\text{ACTM}}$, $X_{\text{a priori}}$, and AK denote the model CH$_4$ concentration, a priori used for GOSAT CH$_4$ retrieval, and GOSAT averaging kernels, respectively.
  - Correlation length used for LETKF experiment is 1700 km.

Column averaged CH$_4$ concentration (inside rectangle in Figure 4) are assimilated. Figure 5 shows that LETKF is able to regenerate true flux in this case too.

(a) Feb 2010

- Summary
  - The sensitivity experiments has been performed on OSSE setting by updating observed changes only into the surface CH$_4$ concentration.
  - The sensitivity experiments also performed by updating the observed change into the full column mimicking the GOSAT observation in the OSSE experiment.
  - Both Fixed multiplicative inflation and relaxation to prior spread (RTPS) method are able to retrieve the CH$_4$ true fluxes well. However, RTPS method performs better than fixed multiplicative inflation.

Figure 4: Column averaged CH$_4$ concentration (ppb) from ACTM collocated with GOSAT observation during Feb 2010.

Figure 5: (a) Same as first and third column of Figure 2(a) but column averaged CH$_4$ concentration is assimilated here. (b) Same as (a) but during July.